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Diffractive η_c photo-and electroproduction with the perturbative QCD Odderon

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Abstract. Using a set of new odderon states, we calculate their contribution to the diffractive η_c photo- and electroproduction process. Compared to previous simple 3-gluon exchange calculations we find an enhancement of about one order of magnitude in the cross section. The t-dependence of the cross section exhibits a dip structure in the small t region.

1 Introduction

The existence of the Odderon [1], the partner of the Pomeron which is odd under charge conjugation C, is an important prediction of perturbative QCD. In the leading order, the Odderon appears as a bound state of three reggeized gluons. Its experimental observation is a strong challenge for the experimentalists. Promising scattering processes where the exchange of the Odderon may be seen include the difference between the $p-p$ and $p-\bar{p}$ cross-sections and the diffractive production of particles with a C-odd exchange, such as photo- and electroproduction of pseudoscalar mesons (PS). From the theoretical point of view, some of the latter processes are of particular interest, namely those where the presence of a large momentum scale provides some justifications for the use of perturbative QCD. This includes, in particular, the diffractive production of charmed pseudoscalar mesons, for example the η_c. Correspondingly, a large amount of literature has been devoted to this class of diffractive processes. For large photon virtualities Q^2, for heavy mass PS mesons (such as η_c), and for large momentum transfers the relevant impact factors for the transition $\gamma(\gamma^*) \to$PS have been be calculated perturbatively [2]. As to the Odderon exchange, first studies have used the simplest form, the exchange of three noninteracting gluons in a $C=-1$ state. Another line of Odderon investigations, pursued by the Heidelberg group, uses a non-perturbative model for the Odderon, based on the idea of a "stochastic QCD vacuum" [3]. Numerical estimates for the cross sections turn out to be very different in these two approaches: the nonperturbative Odderon models tend to give substantially larger cross sections [3, 2,4]. Evidently, to guide the experimental search for signatures of the Odderon we have to clarify these discrepancies. In this paper we follow the perturbative approach and make use of the recently discoverd new Odderon solution.

The perturbative QCD Odderon has a rather long history. After several variational studies [5,6] a first analytic solution to the Odderon equation was constructed by Janik and Wosiek [7] and verified by Braun et al [8]. It belongs to the lowest non-zero eigenvalue of the conformal integral of motion, Q_3. This solution has an intercept slightly below unity and, most important, vanishes if two of the three gluons are at the same point. This property leads to the disappointing result that this solution cannot couple to the perturbative $\gamma \to$PS vertex and therefore is irrelevant for photo and electroproduction of PS mesons. It may, however, play its role in purely hadronic processes, such as pp or $p\bar{p}$ scattering. Recently a new solution for a bound state of three reggeized gluons with the Odderon quantum numbers has been found [9], which is quite different from the previous one. It corresponds to $Q_3=0$, has intercept unity, and, most important, it does couple to the $\gamma \to$PS transition vertex. The structure of the wave function of this solution is rather peculiar, so that one may expect substantial changes compared to the exchange of three noninteracting gluons. The purpose of this paper is to perform an analytic and numerical study of the exchange of this new odderon solution for the diffractive η_c photo- and electroproduction. We follow the approach of [2], by replacing the three noninteracting gluons by the new Odderon state. As the main results, our cross sections are an order of magnitude larger than those of [2,

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4]; also, the t-dependence of our cross section exhibits an interesting dip structure which is not present in the case of noninteracting gluons.

2 The perturbative QCD Odderon which couples to γ-PS transition

Let us first briefly recapitulate the main properties of the new odderon solution. For details we refer to [9]. We remind that this solution has been shown [10] to be a particular case of a more general class of solutions: in the large N_c-limit there exist relations between eigenstates of different number of reggeized gluons, which connect solutions of different symmetry properties, and the Odderon solution is the simplest case that satisfies these relations.

In momentum space the Odderon wave function Ψ is constructed from the known Pomeron solutions $E^{(\nu,n)}$ [11], which have the eigenvalues

$$\chi(\nu,n)=\bar{\alpha}_s\left(2\psi(1)-\psi\left(\frac{1+|n|}{2}+i\nu\right)\right.$$
$$\left.-\psi\left(\frac{1+|n|}{2}-i\nu\right)\right),$$
$$\bar{\alpha}_s=\frac{N_c\alpha_s}{\pi}. \quad (1)$$

One proves that

$$\Psi^{(\nu,n)}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)=c(\nu,n)\sum_{(123)}\frac{(\boldsymbol{k}_1+\boldsymbol{k}_2)^2}{\boldsymbol{k}_1^2\boldsymbol{k}_2^2}$$
$$\times E^{(\nu,n)}(\boldsymbol{k}_1+\boldsymbol{k}_2,\boldsymbol{k}_3), \quad (2)$$

indeed satisfies the Odderon equation and has the same intercept (1), provided the Pomeron wave function is odd under the interchange of its arguments. This restricts the values of n to odd numbers. The normalization factor c in (2) can be chosen in such a way that the Odderon wave function will have the same norm as the Pomeron function

$$\langle\tilde{\Psi}^{(\nu,n)}|\Psi^{(\nu',n')}\rangle=\langle\tilde{E}^{(\nu,n)}|E^{(\nu',n')}\rangle=w(\nu,n)\delta(\nu-\nu')\delta_{nn'}, \quad (3)$$

where the $w(\nu,n)$ are known [12]. In (3) the scalar product is defined as the integral of the two wave functions in momentum space, where the bra-vector has to be amputated (marked by a tilda). Condition (3) leads to

$$c(\nu,n)=\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{g_s^2N_c}{-3\chi(\nu,n)}}\,. \quad (4)$$

When including in (2) the colour structure d_{abc} one should change the normalization by a factor $\sqrt{N_c}/\sqrt{(N_c^2-4)(N_c^2-1)}$.

In order to construct the full Green function we clearly need to know the complete set of solutions of the Odderon equation. At the moment we only have the symmetric solutions of [7] and the new solutions (2). The former are orthogonal to the photon impact factor and so irrelevant for our problem. At present we do not know if any other solution exists, apart from (2), which couples to the $\gamma-PS$ transition vertex. So our results are strictly speaking restricted to the contribution of the exchange of the Odderon states with the wave function (2). Normalizing the Green function to reduce in the small coupling limit ($\alpha_s\to 0$) to

$$\frac{1}{\boldsymbol{k}_1^2\boldsymbol{k}_2^2\boldsymbol{k}_3^2}\delta(\boldsymbol{k}_1-\boldsymbol{k}_1')\delta(\boldsymbol{k}_2-\boldsymbol{k}_2')\,,$$

and having in mind (3) we find the part of the Green function corresponding to (2) in the form:

$$G_3(y|\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3|\boldsymbol{k}_1',\boldsymbol{k}_2',\boldsymbol{k}_3')$$
$$=\sum_{\text{odd }n}\int_{-\infty}^{+\infty}d\nu e^{y\,\chi(\nu,n)}$$
$$\times\frac{(2\pi)^2(\nu^2+n^2/4)}{[\nu^2+(n-1)^2/4][\nu^2+(n+1)^2/4]}$$
$$\times\Psi^{(\nu,n)}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\Psi^{(\nu,n)^*}(\boldsymbol{k}_1',\boldsymbol{k}_2',\boldsymbol{k}_3')\,. \quad (5)$$

3 The BFKL function in the momentum space

In this section we present the BFKL Pomeron eigenstates in the momentum representation, which we need in order to construct the Green's function in (5). This wave function is well known in the coordinate space [12] where its form is dictated by conformal invariance

$$E^{(h,\bar{h})}(\boldsymbol{r}_{10},\boldsymbol{r}_{20})=\left(\frac{r_{12}}{r_{10}r_{20}}\right)^h\left(\frac{\bar{r}_{12}}{\bar{r}_{10}\bar{r}_{20}}\right)^{\bar{h}}, \quad (6)$$

where $\boldsymbol{r}_{10}=\boldsymbol{r}_1-\boldsymbol{r}_0$ etc, $h=(1+n)/2+i\nu$, $\bar{h}=(1-n)/2+i\nu$, and the standard complex notation for the two-dimensional vector is used on the right-hand side. By Fourier transforming to momentum space one finds (see Appendix A)

$$\tilde{E}_{h\bar{h}}(\boldsymbol{k}_1,\boldsymbol{k}_2)=\tilde{E}^A_{h\bar{h}}(\boldsymbol{k}_1,\boldsymbol{k}_2)+\tilde{E}^\delta_{h\bar{h}}(\boldsymbol{k}_1,\boldsymbol{k}_2), \quad (7)$$

where the first term denotes the analytic contribution, and the second one stands for the δ-function terms. The analytic part is given by

$$\tilde{E}^A_{h\bar{h}}(\boldsymbol{k}_1,\boldsymbol{k}_2)=C\Big(X(\boldsymbol{k}_1,\boldsymbol{k}_2)+(-1)^nX(\boldsymbol{k}_2,\boldsymbol{k}_1)\Big), \quad (8)$$

where $h=(1+n)/2+i\nu$ and $\bar{h}=(1-n)/2+i\nu$ are the conformal weights. The coefficient C is given by

$$C=\frac{(-i)^n}{(4\pi)^2}h\bar{h}(1-h)(1-\bar{h})\Gamma(1-h)\Gamma(1-\bar{h}). \quad (9)$$

The expression for X in complex notation is given in terms of the hypergeometric functions

$$X(\boldsymbol{k}_1,\boldsymbol{k}_2)=\left(\frac{k_1}{2}\right)^{\bar{h}-2}\left(\frac{\bar{k}_2}{2}\right)^{h-2}F\left(1-h,2-h;2;-\frac{\bar{k}_1}{\bar{k}_2}\right)$$
$$\times F\left(1-\bar{h},2-\bar{h};2;-\frac{k_2}{k_1}\right). \quad (10)$$

The δ-function part is

$$\tilde{E}^{\delta}_{h\bar{h}}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \left[\delta^{(2)}(\boldsymbol{k}_1) + (-1)^n \delta^{(2)}(\boldsymbol{k}_2)\right] \frac{i^n}{2\pi} 2^{1-h-\bar{h}} \times \frac{\Gamma(1-\bar{h})}{\Gamma(h)} q^{\bar{h}-1} q^{*\,h-1}\,, \quad \boldsymbol{q} = \boldsymbol{k}_1 + \boldsymbol{k}_2. \tag{11}$$

Let us note that, when the Pomeron couples to an impact factor of a colourless object, δ-function terms in the pomeron wave function do not give any contribution. However in our calculations it turns out that these terms play an important role.

In order to calculate the couplings of the exchanged Odderon to the proton and to the $\gamma \to \eta_c$ impact factors we need to know the Pomeron wave function $E^{(\nu,n)}$ in momentum space for $n = \pm 1$ and around the value $\nu = 0$. We start with the analytic part; we need to look at (10) which, for $n = 1$, reads

$$X(\boldsymbol{k}_1,\boldsymbol{k}_2) = \left(\frac{k_1}{2}\right)^{i\nu-2} \left(\frac{\bar{k}_2}{2}\right)^{i\nu-1} F\left(-i\nu, 1-i\nu; 2; -\frac{\bar{k}_1}{\bar{k}_2}\right) \times F\left(1-i\nu, 2-i\nu; 2; -\frac{k_2}{k_1}\right), \tag{12}$$

and we perform a Taylor expansion around the point $\nu = 0$. The complicated expression (8) is drastically simplified at small values of ν. In lowest order it is linear in ν:

$$E^A_1(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{\nu}{2\pi^2 q}\left(\frac{1}{k_1\bar{k}_2} - \frac{1}{k_2\bar{k}_1}\right). \tag{13}$$

This function is odd in the azimuthal angle (i.e. antisymmetric under $\varphi \to \varphi + \pi$). So it is orthogonal to the two impact factors which are azimuthally even. For this reason a non-zero contribution only comes from the terms quadratic in ν. Omitting those with the same structure as (13) we find

$$E^A_2(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{i\nu^2}{2\pi^2 q}\left[\frac{1}{\boldsymbol{k}_2^2}\ln \boldsymbol{k}_1^2 - \frac{1}{\boldsymbol{k}_1^2}\ln \boldsymbol{k}_2^2 + \left(\frac{1}{\boldsymbol{k}_1^2} - \frac{1}{\boldsymbol{k}_2^2}\right) \times \ln q + \left(\frac{1}{\bar{k}_2\boldsymbol{k}_1^2} - \frac{1}{\bar{k}_1\boldsymbol{k}_2^2}\right)\bar{q}\ln\bar{q}\right]. \tag{14}$$

The δ-function part is finite for $\nu \to 0$:

$$E^{\delta}_0(\boldsymbol{k}_1,\boldsymbol{k}_2) = \left[\delta^{(2)}(\boldsymbol{k}_1) - \delta^{(2)}(\boldsymbol{k}_2)\right]\frac{i}{2\pi}\frac{1}{q}. \tag{15}$$

The constant C in (9) (for $n = 1$) becomes:

$$C = \frac{1}{(4\pi)^2}\nu(1+\nu^2)\Gamma^2(1-i\nu). \tag{16}$$

To construct the Green's function one has also to consider

$$\chi(\nu,\pm 1) = -\nu^2\, 2\bar{\alpha}_s\zeta(3) + \mathcal{O}(\nu^4) \tag{17}$$

and the ν-dependence in the denominators of the integration measure in (5):

$$[\nu^2 + (n-1)^2/4][\nu^2 + (n+1)^2/4] = \nu^2(1+\nu^2)\,. \tag{18}$$

4 The transition amplitude

We study the process of the diffractive photo- or electroproduction of η_c on the proton: $\gamma^* p \to \eta_c p$. It will be assumed that the proton remains intact, although it would be rather easy to include also its low-lying excitations. The differential cross-section is given by the formula

$$\frac{d\sigma}{dt}(\gamma(\gamma^*) + p \to \eta_c + p) = \frac{1}{16\pi s^2}\,\frac{1}{2}\sum_{i=1}^{2}|A^i|^2, \tag{19}$$

where A^i, $i = 1, 2$ is the photoproduction amplitude for a given transverse polarization i of the photon. The electroproduction cross-section can be obtained from (19) in a trivial manner (see [2]). The photoproduction amplitude A^i is given by a convolution of the two impact factors, Φ_p and Φ^i_γ, for the proton and for the $\gamma \to \eta_c$ transition, resp., with the Odderon Green function:

$$A^i = \frac{s}{32}\frac{5}{6}\frac{1}{3!}\frac{1}{(2\pi)^8}\langle\Phi^i_\gamma|G_3|\Phi_p\rangle. \tag{20}$$

We shall assume that the c.m. energy squared, s, is much greater than the scales in the transition vertex: $s \gg Q^2$, m_c^2, t, t being the four momentum squared carried by the Odderon. Using the definition of [2], namely $x = (m^2_{\eta_c} + Q^2)/(s + Q^2)$ and $y = \log 1/x$, we are in the low-x limit $x \ll 1$.

The matrix element on the rhs of (20) involves the integration over ν coming from the Green function (5). This integration can be done in the saddle point approximation, since $y \gg 1$. The leading contribution will obviously come from the smallest values of $|n|$, i.e. $n = \pm 1$ and small ν, when the integral acquires a form

$$\int d\nu e^{-\nu^2\beta\, y} I(\nu), \quad \beta = 2\bar{\alpha}_s\zeta(3) \tag{21}$$

and $I(\nu)$ denotes the nonexponential part of the integrand. Since we expect the dominant contribution to the cross section to come from the kinematical region where t is not large, there is one more large momentum scale in the problem, apart from the energy $\sqrt{s}$, namely, $M = \sqrt{Q^2 + 4m_c^2}$, which provides some basis for the use of perturbation theory. In principle, the position of the saddle point ν_s depends upon the relation between these two large scales. However in our kinematical region of small x, M is much smaller than $\sqrt{s}$, so that ν_s is close to zero. As a result, we have to calculate the matrix elements of the two impact factors with the Odderon wave function with $|n| = 1$ and small ν.

The impact factor corresponding to the transition $\gamma \to \eta_c$ can be calculated perturbatively. Referring the reader to the original paper [2] for the details, we only quote the final result:

$$\Phi^i_\gamma = b\,\epsilon_{ij}\frac{q_j}{\boldsymbol{q}^2}\left(\sum_{(123)}\frac{(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3)\cdot\boldsymbol{q}}{Q^2 + 4m_c^2 + (\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3)^2} - \frac{\boldsymbol{q}^2}{Q^2 + 4m_c^2 + \boldsymbol{q}^2}\right) \tag{22}$$

with $\boldsymbol{q} = \boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3$ and

$$b = \frac{16}{\pi} e_c g_s^3 \frac{1}{2} m_{\eta_c} b_0. \quad (23)$$

Here $e_c = (2/3)e$ is the electric charge of the charmed quark, and g_s is the strong coupling constant. The constant b_0 can be determined from the known radiative width $\Gamma(\eta_c \to \gamma\gamma)$= 7 KeV:

$$b_0 = \frac{16\pi^3}{3e_c^2}\sqrt{\frac{\pi\Gamma}{m_{\eta_c}}}. \quad (24)$$

Following [9] we denote

$$\varphi(\boldsymbol{k}, \boldsymbol{k}') = \frac{(\boldsymbol{k} - \boldsymbol{k}') \cdot (\boldsymbol{k} + \boldsymbol{k}')}{Q^2 + 4m_c^2 + (\boldsymbol{k} - \boldsymbol{k}')^2}. \quad (25)$$

Using the form (2) of the Odderon wave function one can show that the convolution of the Odderon wave function Ψ with the impact factor Φ_γ for the transition $\gamma \to \eta_c$ reduces to a convolution of the function φ and the Pomeron function E [9]:

$$\begin{aligned}\langle \Phi^i_{\gamma\to\eta_c} | \Psi^{(\nu,n)} \rangle &= b\epsilon_{ij} \frac{q_j}{\boldsymbol{q}^2} \frac{1}{c(\nu,n)} \int d^2\boldsymbol{k}\, \varphi(\boldsymbol{k}, \boldsymbol{q} - \boldsymbol{k}) \\ &\quad \times E^{(\nu,n)}(\boldsymbol{k}, \boldsymbol{q} - \boldsymbol{k}) \\ &\equiv -b\epsilon_{ij} \frac{q_j}{\boldsymbol{q}^2} \frac{1}{c} \frac{1}{M} |t|^{i\nu} \frac{i}{\pi} V_\gamma^{(\nu,n)}\left(\frac{|t|}{M^2}\right), \quad (26)\end{aligned}$$

where the Pomeron function is given in (7), and we have rescaled the momenta in the integrand by $q = \sqrt{|t|}$.

Note that (26) is sensitive to the δ-function term present in the Pomeron wave function. In fact in [9] (26) has been proven by explicitly using the Pomeron wave function in the coordinate representation, which does contain such terms.

We could have chosen to calculate $\langle \Phi^i_{\gamma\to\eta_c} | \Psi^{(\nu,n)} \rangle$ in a different way, directly from (2):

$$\langle \Phi^i_{\gamma\to\eta_c} | \Psi^{(\nu,n)} \rangle = 3c \int d^2\boldsymbol{l}\, E^{*\,(\nu,n)}(\boldsymbol{l}, \boldsymbol{q} - \boldsymbol{l}) g(\boldsymbol{l}), \quad (27)$$

where function $g(\boldsymbol{l})$ is defined by

$$g(\boldsymbol{l}) = \int d^2\boldsymbol{k}\, \frac{\boldsymbol{l}^2}{\boldsymbol{k}^2(\boldsymbol{l} - \boldsymbol{k})^2} \Phi^i_\gamma(\boldsymbol{k}, \boldsymbol{l} - \boldsymbol{k}, \boldsymbol{q} - \boldsymbol{l})\,, \quad (28)$$

the factor 3 comes from the symmetry of the impact factor and $\boldsymbol{k}_1 = \boldsymbol{k}$, $\boldsymbol{k}_2 = \boldsymbol{l} - \boldsymbol{k}$ and $\boldsymbol{k}_3 = \boldsymbol{q} - \boldsymbol{l}$.

Now the δ-function terms in E give no contribution. Indeed, $g(\boldsymbol{0}) = 0$ because of the $\boldsymbol{l}^2$ factor present in (28), and $g(\boldsymbol{q}) = 0$ as the impact factor vanishes for $\boldsymbol{k}_3 = \boldsymbol{0}$. As a result, in the calculation of (27), one may freely add or subtract such δ-function pieces in the Pomeron wave function [13]. At $\nu = \pm 1$ and small ν factor c behavies as $1/\nu$ (see (4) and (17)). Therefore, to find the $\gamma \to$PS form-factor in the lowest (first) order in ν one needs to know the BFKL function $E^{(\nu,\pm1)}$ in the second order if one uses (27), but only in the zeroth order if one uses (26). One can show that both ways lead to the same answer. However, (27) needs numerical computation, and (26) gives the result in a trivial manner due to the simple structure of the zeroth order BFKL function (15). At $|n| = 1$ and $\nu \to 0$ we find

$$V_\gamma^{(0,\pm1)}\left(\frac{t}{M^2}\right) = \frac{\sqrt{|t|/M^2}}{1 + |t|/M^2}\,. \quad (29)$$

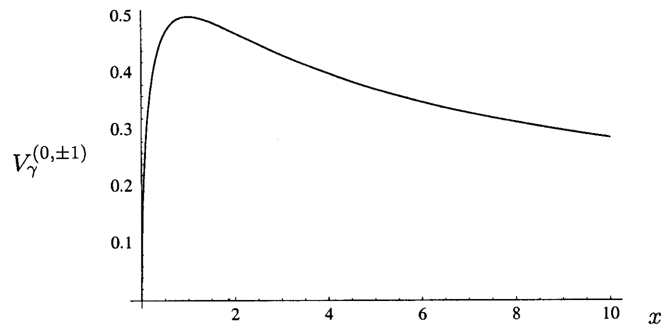

Fig. 1. Numerical result for the coupling of the Odderon to the $\gamma^* \to \eta_c$ impact factor (defined in (26)), as a function of the scaled variable $x = |t|/(Q^2 + 4m_c^2)$

In Fig. 1 we show a plot of this function in the region $0 \le |t|/M^2 \le 10$. We have verified by numerical computation that, starting from (27) and using for $E^{*\,(\nu,n)}$ only the analytic part, E_2^A, we get exactly the same answer for $\nu \to 0$.

The proton impact factor is non-perturbative. We use the parametrization proposed in [2]:

$$\Phi_p = d\Big[F(\boldsymbol{q}, 0, 0) - \sum_{i=1}^{3} F(\boldsymbol{k}_i, \boldsymbol{q} - \boldsymbol{k}_i, 0) + 2F(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\Big], \quad (30)$$

with

$$\begin{aligned}&F(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\ &\quad = \frac{2a^2}{2a^2 + (\boldsymbol{k}_1 - \boldsymbol{k}_2)^2 + (\boldsymbol{k}_2 - \boldsymbol{k}_3)^2 + (\boldsymbol{k}_3 - \boldsymbol{k}_1)^2}, \quad (31)\end{aligned}$$

$d = 8(2\pi)^2\bar{g}^3$ and the scale parameter $a = m_\rho/2$. From the comparison with the two gluon exchange model for hadronic cross-sections the authors of [2] estimate $\bar{g}^2/(4\pi)$ = 1. The impact factor (30) satisfies the basic requirement that it vanishes when any of the three gluon momenta goes to zero. The calculation of the scalar product of the Odderon wave function with the proton impact factor is more cumbersome, since it amounts to the integration over the three-gluon phase space. Using the symmetry of the proton impact factor and the explicit form of the Odderon wave function (2) one finds

$$\begin{aligned}\langle \Phi_p | \Psi^{(\nu,n)} \rangle &= 3c\, d \int d^2\boldsymbol{l}\, E^{*\,(\nu,n)}(\boldsymbol{l}, \boldsymbol{q} - \boldsymbol{l}) f(\boldsymbol{l}) \\ &= c\, d \frac{1}{(2a^2)^{1/2}} |t|^{-i\nu} \frac{(i\nu^2)^*}{2\pi^2} V_p^{(\nu,n)}\left(\frac{|t|}{2a^2}\right), \quad (32)\end{aligned}$$

where

$$f(\boldsymbol{l}) = \int d^2\boldsymbol{k} \frac{\boldsymbol{l}^2}{\boldsymbol{k}^2(\boldsymbol{l}-\boldsymbol{k})^2} \Bigg[F(\boldsymbol{q},0,0) - \sum_{j=1}^{3} F(\boldsymbol{k}_j, \boldsymbol{q}-\boldsymbol{k}_j, 0) + 2F(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \Bigg] \quad (33)$$

and, as before, $\boldsymbol{k}_1 = \boldsymbol{k}$, $\boldsymbol{k}_2 = \boldsymbol{l}-\boldsymbol{k}$ and $\boldsymbol{k}_3 = \boldsymbol{q}-\boldsymbol{l}$. To obtain the last expression in (32) we have again rescaled the momenta in the integral with respect to q and extracted the factor $i\nu^2/2\pi^2$ having in mind that at small ν the contribution to (32) starts with the second order term of the BFKL function (14). Note that the integral (33) is infrared finite, since the square bracket vanishes if any of the gluon momenta goes to zero. However, individual terms inside the square bracket are infrared divergent. Clearly, as in (27), only the analytic part of the Pomeron function E^A contributes to (32).

Let us consider the rhs of (33). Two of the five terms are simple, since the F functions do not depend on the integration variable. They give

$$f_1 = \Big(F(\boldsymbol{q},0,0) - F(\boldsymbol{q}-\boldsymbol{l},\boldsymbol{l},0)\Big) \int d^2\boldsymbol{k} \frac{\boldsymbol{l}^2}{\boldsymbol{k}^2(\boldsymbol{l}-\boldsymbol{k})^2} = \Big(F(\boldsymbol{q},0,0) - F(\boldsymbol{q}-\boldsymbol{l},\boldsymbol{l},0)\Big) 2\pi \ln \frac{\boldsymbol{l}^2}{m^2}. \quad (34)$$

We have introduced here a mass m as an infrared regulator. The two remaining terms in the sum inside the square brackets in (33) give identical contributions. Their sum is given by

$$f_2 = -\frac{2}{3}a^2 \int d^2\boldsymbol{k} \frac{\boldsymbol{l}^2}{\boldsymbol{k}^2(\boldsymbol{l}-\boldsymbol{k})^2} \frac{1}{(\boldsymbol{k}-\boldsymbol{q}/2)^2+\epsilon}, \quad (35)$$

where $\epsilon = \frac{1}{12}\boldsymbol{q}^2 + \frac{1}{3}a^2$. The last term in the sum gives:

$$f_3 = \frac{2}{3}a^2 \int d^2\boldsymbol{k} \frac{\boldsymbol{l}^2}{\boldsymbol{k}^2(\boldsymbol{l}-\boldsymbol{k})^2} \frac{1}{(\boldsymbol{k}-\boldsymbol{l}/2)^2+\delta}. \quad (36)$$

where $\delta = \boldsymbol{l}^2 - \boldsymbol{l}\boldsymbol{q} + (\boldsymbol{q}^2+a^2)/3$. Both integrals (35) and (36) thus reduce to a general two-dimensional triangle diagram

$$I = \int d^2\boldsymbol{k} \frac{1}{(\boldsymbol{k}^2+m^2)[(\boldsymbol{k}-\boldsymbol{l})^2+m^2][(\boldsymbol{k}-\boldsymbol{p})^2+b^2]}. \quad (37)$$

Using the standard Feynman parametrization this integral can be transformed into an one-dimensional one which, after separating the infrared divergent contributions, can be done numerically. Some details about this procedure are discussed in Appendix B.

The resulting function $f(\boldsymbol{l}) = f_1 + f_2 + f_3$ is used to calculate the integral (32). Here only the analytic part, E_2^A, contributes. The calculation has been done numerically. We present the results for $V_p^{(0,\pm1)}$ in Fig. 2. One observes that V_p changes sign at $|t| \approx 0.07$ GeV^2. As a consequence, the cross section will vanish at this point. Of course, this property is literally true only in the limit of asymptotically large energies where one can neglect the contributions of all other states $\nu \neq 0$ and $|n| > 1$.

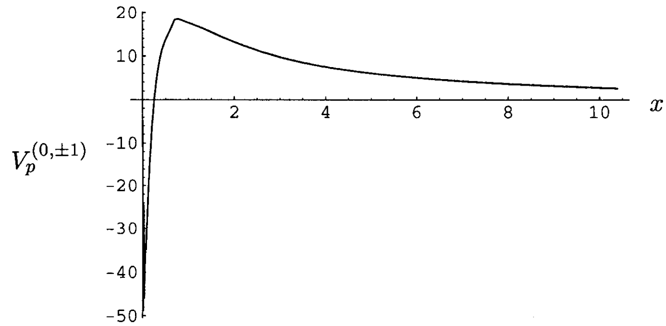

Fig. 2. Numerical results for the coupling of the Odderon to the proton, defined in (32), as a function of the scaled variable $x = |t|/2a^2$

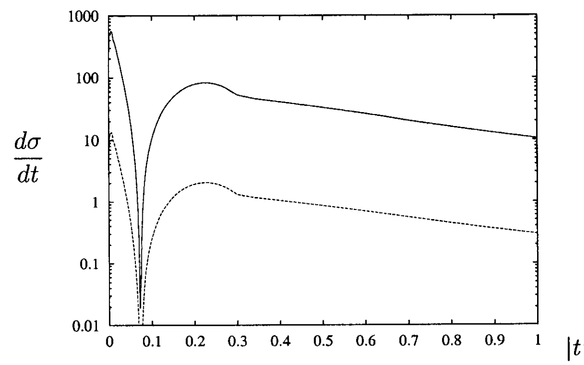

Fig. 3. The differential cross sections (in pb / GeV^2). The upper curve refers to $Q^2 = 0$

5 Numerical results and discussion

To find the final cross-sections from (19) and (20), we do the saddle point integration over ν, then take the square module of the amplitude (20) and do the sum over the polarizations in (19). The latter step provides a factor $1/|t|$ (from the prefactors in (22)). The normalization factors $c(\nu, n)$ of the Odderon solution which are contained in the Green's function (5) cancel when the scalar product (26) of the Odderon wave function and the photon impact factor is computed. Collecting all pieces of our cross section formula we find

$$\frac{d\sigma}{dt}(\gamma(\gamma^*) + p \to \eta_c + p) = \frac{2^4 \cdot 5^2}{3^7} \frac{1}{(2\pi)^8} \frac{\alpha_{em}\alpha_s^2 b_0^2}{\zeta(3) y} \frac{m_{\eta_c}^2}{(Q^2+4m_c^2)2a^2} \times \frac{1}{|t|} \left| V_\gamma^{(0,\pm1)}(t) \right|^2 \left| V_p^{(0,\pm1)}(t) \right|^2. \quad (38)$$

The differential cross sections for the two cases $Q^2 = 0$ and 25 GeV^2 and $\sqrt{s} \approx 300$ GeV are shown in Fig.3. We have taken α_s at the scale $m_c^2 + Q^2$ (in [2] at $Q^2 = 0$ the scale was m_c^2). Independently of Q^2 the cross sections show a dip at small $|t| \approx 0.07$ GeV and a maximum at $|t| \approx 0.22$ GeV (roughly of the order of $(m_\rho/2)^2$. The dip comes from the zero present in the Odderon-proton coupling (Fig. 2). Its origin seems to be related to the symmetry properties of the Odderon solution: as it can be seen from (2), the Odderon wave function is a sum of three terms, each of which contains an antisymmetric Pomeron eigenfunction. A similar structure is present in the photon impact factor (22), whereas the proton impact factor (30) is completely symmetric. The convolution of the Odderon wave function with the photon impact factor has no zero in t, whereas the convolution with the proton leads to such a zero. Whether this feature is an artifact of the simple model for the proton impact factor that we have used, or whether it represents a general property of the Odderon-proton coupling we do not know. In our calculation, this dip is present in the leading high energy approximation; it may be that at finite energies (e.g. at HERA) the dip is (partially) filled by the exchange of nonleading Odderon states.

At $t = 0$ the cross section vanishes, as in the case of a simple three gluon exchange. As one can see from Fig. 3, this happens at quite small t and cannot be seen in the figure. A more detailed analysis of the behaviour in the region of very small t requires, probably, a more accurate evaluation of the ν-integral in the 3-gluon Green's function. It would, however, not affect too much the dip structure or the value of the integrated cross section.

For the integrated cross sections we find 50 pb and 1.3 pb at $Q^2 = 0$ and 25 GeV^2, respectively. Compared to the value of 11 pb predicted for $Q^2 = 0$ with a simple three-gluon exchange [2] we find an enhancement of about 5 times. For $Q^2 = 25$ GeV^2 the differential cross sections in [2] seem to indicate that one can simply scale the $Q^2 = 0$ cross section by 0.01 and obtain 0.1pb. This implies that our cross-section is an order of magnitude larger. Note that compared to the simple three gluon exchange, we have a (weak) logarithmic suppression with energy. So the obtained enhancement effect is totally due to the coupling of our Odderon wave function to the impact factors.

In conclusion, by comparing the exchange of three non-interacting gluons with the exchange of the new odderon solution we find that the interaction between the exchanged gluons leads to a significant change in the scattering cross section. However, despite this improvement in accuracy, our numerical estimate of the η_c cross section still suffers from a few theoretical uncertainties, in particular due to the Odderon-proton coupling. We believe that both the structure of the vertex and its overall normalization should be checked more carefully. As a possible strategy, one might study the exchange of the Odderon in pp and $p\bar{p}$ scattering at large t where the use of perturbative QCD can be justified. Using the same model [2], a comparison with experimental data on the difference of pp and $p\bar{p}$ scattering fixes the overall normalization of the Odderon proton coupling. As to the general momentum structure of the vertex, the most sensitive test is the t-dependence in the small-t region: the presence of a dip would support both the structure of the Odderon-proton vertex and of the Odderon state used in our calculation.

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Appendix A: A momentum space representation of the Pomeron wave function in the non-forward direction

The Fourier transform of the function $E^{(h,\bar{h})}$ given by (6) is defined by

$$\tilde{E}_{h\bar{h}}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \int \frac{d^2\boldsymbol{r}_1}{(2\pi)^2} \frac{d^2\boldsymbol{r}_2}{(2\pi)^2} \left(\frac{r_{12}}{r_1 r_2}\right)^h \left(\frac{r_{12}^*}{r_1^* r_2^*}\right)^{\bar{h}} \times e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}_1 + \boldsymbol{k}_2\cdot\boldsymbol{r}_2)} . \quad (39)$$

The non exponential part of the integrand in (39) becomes constant as $|\boldsymbol{r}_i| \to \infty$; $i = 1$ or 2 and therefore contains terms proportional to $\delta^2(\boldsymbol{k}_1)$ or $\delta^2(\boldsymbol{k}_2)$. Therefore one expects to find

$$\tilde{E}_{h\bar{h}}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \tilde{E}^A_{h\bar{h}}(\boldsymbol{k}_1, \boldsymbol{k}_2) + \tilde{E}^\delta_{h\bar{h}}(\boldsymbol{k}_1, \boldsymbol{k}_2), \quad (40)$$

where the first term denotes the analytic contribution and the second the δ-like one.

We shall at first compute the analytic part of the Fourier transform. Since we are dealing with a distribution, it is convenient to consider a new, regularized, object

$$\tilde{E}^{(reg)}_{h\bar{h},h_3\bar{h}_3}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \int \frac{d^2\boldsymbol{r}_1}{(2\pi)^2} \frac{d^2\boldsymbol{r}_2}{(2\pi)^2} \frac{(r_{12})^{h_3}}{(r_1 r_2)^h} \frac{(r_{12}^*)^{\bar{h}_3}}{(r_1^* r_2^*)^{\bar{h}}} \times e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}_1 + \boldsymbol{k}_2\cdot\boldsymbol{r}_2)} , \quad (41)$$

with an independent conformal weigth h_3 for the r_{12}-terms. The integral (41) is well defined for $\Re(h + \bar{h}) < 2$, $\Re(h_3 + \bar{h}_3) > -2$ and $\Re(h + \bar{h} - h_3 - \bar{h}_3) > 0$. Strictly speaking, the last inequality holds, for example, for the $\boldsymbol{k}_2 \neq \boldsymbol{0}$ case. When $\boldsymbol{k}_2 = \boldsymbol{0}$ one needs $\Re(h + \bar{h} - h_3 - \bar{h}_3) > 2$. In the calculation of the scalar product of the Pomeron function with test functions which vanishe at the points $\boldsymbol{k}_1 = 0$ or $\boldsymbol{k}_2 = 0$ we shall, therefore, consider the following prescription: $\langle \Phi | E_{h\bar{h}} \rangle = \lim_{h_3 \to h} \langle \Phi | E_{h\bar{h},h_3\bar{h}_3} \rangle = \langle \Phi | \lim_{h_3 \to h} E_{h\bar{h},h_3\bar{h}_3} \rangle$. This means that we shall be able to extract the analytic part of (39).

Introducing, for both external gluons of the Pomeron wave function, the complex variables

$$r = r_x + ir_y \equiv x + iy \quad \kappa = \frac{k_x - ik_y}{2}$$
$$r^* = r_x - ir_y \equiv x - iy \quad \kappa^* = \frac{k_x + ik_y}{2}, \quad (42)$$

and remembering that $h - \bar{h} = n$ and $h_3 - \bar{h}_3 = n_3$, we get a double integral in the complex plane:

$$\tilde{E}^{(reg)}_{h\bar{h},h_3\bar{h}_3}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \int \frac{d^{\mathcal{C}}\boldsymbol{r}_1 d^{\mathcal{C}}\boldsymbol{r}_2}{(2\pi)^4} \left|r_1^2\right|^{-h} \left|r_2^2\right|^{-h} \left|r_{12}^2\right|^{h_3} r_1^{*\,n} \times r_2^{*\,n} r_{12}^{*\,-n_3} e^{i(\kappa_1 r_1 + \kappa_1^* r_1^* + \kappa_2 r_2 + \kappa_2^* r_2^*)} . \quad (43)$$

By using the integral representation

$$x^{-u} = \frac{1}{\Gamma(u)} \int_0^\infty d\alpha\, \alpha^{u-1} e^{-\alpha x} \qquad (\Re(x) > 0\ ,\ \Re(u) > 0) \tag{44}$$

we arrive at the representation

$$\begin{aligned} E^{(reg)}_{h\bar{h}}(\kappa_1,\kappa_2) &= \frac{1}{\Gamma^2(h)\Gamma(-h_3)} \int \frac{d^{\mathcal{C}}\boldsymbol{r}_1 d^{\mathcal{C}}\boldsymbol{r}_2}{(2\pi)^4} (r_1^* r_2^*)^n \\ &\times r_{12}^{*\,-n_3} e^{i(\kappa_1 r_1 + \kappa_1^* r_1^* + \kappa_2 r_2 + \kappa_2^* r_2^*)} \\ &\times \int_0^\infty d\alpha_1 d\alpha_2 d\alpha_3\, (\alpha_1\alpha_2)^{h-1} \alpha_3^{-h_3-1} \\ &\times e^{-(\alpha_1 r_1 r_1^* + \alpha_2 r_2 r_2^* + \alpha_3 r_{12} r_{12}^*)}\ , \end{aligned} \tag{45}$$

where, in order to fulfill the restrictions in (44), we require

$$\Re(h) > 0\ ,\ \Re(h_3) < 0 \iff n > -1\ ,\ n_3 < -1\ . \tag{46}$$

The next step is to perform a suitable change of variables in order to do the spatial integrations. Thanks to the holomorphic separability of the integrand, the natural choice would be to use r_j and r_j^* as independent integration variables. This can practically be achieved in the following way: first of all note that the whole integrand, regarded as a function of (x_1, y_1, x_2, y_2), is analytic. Therefore, we can analytically continue to complex values of, say, y_1 and y_2, and rotate the respective integration paths by negative angles. This can be done by putting $y = e^{-i\theta} w : w \in \mathcal{R}$ for both $j = 1, 2$, and by simultaneously rotating the α-integrations in such a way that $\alpha = e^{i\phi}\beta : \beta \in \mathcal{R}$. The generic exponential in the second line in (45) becomes

$$\begin{aligned} e^{-\alpha r r^*} &= e^{-\alpha(x^2+y^2)} \\ &= e^{-\beta e^{i\phi}(x^2 + e^{-2i\theta} w^2)} \\ &= e^{-\beta x^2 e^{i\phi}} e^{-\beta w^2 e^{i(\phi - 2\theta)}}, \end{aligned} \tag{47}$$

which does not grow for large x and w provided $|\phi| \le \pi/2\ ,\ |\phi - 2\theta| \le \pi/2$. Choosing the extreme case $\phi = \theta = \pi/2$ we set $y = -iw\ ,\ \alpha = i\beta$ and perform the change of variables

$$\rho_j = x_j + w_j\ ,\ \bar{\rho}_j = x_j - w_j \qquad (j = 1, 2)\ . \tag{48}$$

A further remark concerns the values of the complex momenta k_j: the exponential in the first line in (45) contains two factors

$$e^{i(k_x x + k_y y)} = e^{i(k_x x - i k_y w)} = e^{i k_x x} e^{k_y w}\ . \tag{49}$$

In order to have a meaningful integral, the values of the y-components of the momenta must be imaginary. Hence we have to introduce a slightly different notation with respect to (42)

$$\kappa = \frac{k_x - i k_y}{2}\ ;\ \bar{\kappa} = \frac{k_x + i k_y}{2} \neq \kappa^*\ ;\ \kappa, \bar{\kappa} \in \mathcal{R} \tag{50}$$

and to consider κ and $\bar{\kappa}$ as independent variables. After this replacement we get

$$\begin{aligned} & E^{(reg)}_{h\bar{h}}(\kappa_1,\kappa_2;\bar{\kappa}_1,\bar{\kappa}_2) \\ &= \frac{(-i/2)^2(2\pi)^{-4}}{\Gamma^2(h)\Gamma(-h_3)} \int_{-\infty}^\infty d\bar{\rho}_1 d\bar{\rho}_2\, (\bar{\rho}_1\bar{\rho}_2)^n \bar{\rho}_{12}^{-n_3} e^{i(\bar{\kappa}_1\bar{\rho}_1 + \bar{\kappa}_2\bar{\rho}_2)} \\ &\times i^{2h-h_3} \int_0^\infty d\beta_1 d\beta_2 d\beta_3\, (\beta_1\beta_2)^{h-1} \beta_3^{-h_3-1} \\ &\times \int_{-\infty}^\infty d\rho_1 d\rho_2\, e^{-i(\beta_1\rho_1\bar{\rho}_1 + \beta_2\rho_2\bar{\rho}_2 + \beta_3\rho_{12}\bar{\rho}_{12} - \kappa_1\rho_1 - \kappa_2\rho_2)}. \end{aligned} \tag{51}$$

The last integral in the above expression provides two delta functions constraining

$$\begin{pmatrix}\kappa_1\\ \kappa_2\end{pmatrix} = \begin{pmatrix}\beta_1+\beta_3 & -\beta_3\\ -\beta_3 & \beta_2+\beta_3\end{pmatrix}\begin{pmatrix}\bar{\rho}_1\\ \bar{\rho}_2\end{pmatrix} \iff$$
$$\begin{pmatrix}\bar{\rho}_1\\ \bar{\rho}_2\end{pmatrix} = \frac{1}{\beta_1\beta_2+\beta_2\beta_3+\beta_3\beta_1}\begin{pmatrix}\beta_2+\beta_3 & \beta_3\\ \beta_3 & \beta_1+\beta_3\end{pmatrix}\begin{pmatrix}\kappa_1\\ \kappa_2\end{pmatrix}.$$

If, for simplicity, we restrict ourselves to positive values of the κ's, it is apparent from the second equality in the above expression that the positivity of the β's forces the $\bar{\rho}$-integrals to contribute only in the positive real half-plane. At this point we split the $\bar{\rho}$-integral into two pieces: the first takes into account the region $\bar{\rho}_2 < \bar{\rho}_1$ and the second the remaining one $\bar{\rho}_2 > \bar{\rho}_1$. The first contribution reads

$$\begin{aligned} & \int_0^\infty d\bar{\rho}_1 \int_0^{\bar{\rho}_1} d\bar{\rho}_2\, (\bar{\rho}_1\bar{\rho}_2)^n \bar{\rho}_{12}^{-n_3} e^{i(\bar{\kappa}_1\bar{\rho}_1+\bar{\kappa}_2\bar{\rho}_2)} \\ &\times \int_0^\infty d\beta_1 d\beta_2 d\beta_3\, (\beta_1\beta_2)^{h-1}\beta_3^{-h_3-1} \\ &\times \delta(\beta_1\bar{\rho}_1 + \beta_3\bar{\rho}_{12} - \kappa_1)\delta(\beta_2\bar{\rho}_2 - \beta_3\bar{\rho}_{12} - \kappa_2). \end{aligned} \tag{52}$$

We can factorize the above expression in the product of two independent integrals by means of the change of variables $\lambda_i = \beta_i\bar{\rho}_i : i = 1, 2, 3\ ,\ (\bar{\rho}_3 \equiv \bar{\rho}_{12})$, which yields

$$\begin{aligned} & \int_0^\infty d\bar{\rho}_1 \int_0^{\bar{\rho}_1} d\bar{\rho}_2\, (\bar{\rho}_1\bar{\rho}_2)^{n-h} \bar{\rho}_{12}^{-n_3+h_3} e^{i(\bar{\kappa}_1\bar{\rho}_1+\bar{\kappa}_2\bar{\rho}_2)} \\ &\times \int_0^\infty d\lambda_1 d\lambda_2 d\lambda_3\, (\lambda_1\lambda_2)^{h-1}\lambda_3^{-h_3-1} \\ &\times \delta(\lambda_1 + \lambda_3 - \kappa_1)\delta(\lambda_2 - \lambda_3 - \kappa_2)\ . \end{aligned} \tag{53}$$

The $\bar{\rho}$-integrals are easily evaluated by setting $x = \bar{\rho}_2/\bar{\rho}_1$, which casts the inner integral into an integral representation of the confluent hypergeometric function ${}_1F_1$ (see, e.g., (13.2.1) of [14]). The outer integral then becomes a Laplace transform of the ${}_1F_1$ times a power (see (7.621.4) of [15]). The result for the first factor of (53) is (remember $n - h = -\bar{h}$)

$$\begin{aligned} & -\, i^{\bar{h}_3 - 2\bar{h}} \frac{\Gamma(1-\bar{h})\Gamma(1+\bar{h}_3)\Gamma(2-2\bar{h}+\bar{h}_3)}{\Gamma(2-\bar{h}+\bar{h}_3)} \bar{\kappa}_1^{2\bar{h}-\bar{h}_3-2} \\ &\times\ {}_2F_1\left(1-\bar{h}, 2-2\bar{h}+\bar{h}_3; 2-\bar{h}+\bar{h}_3; -\frac{\bar{\kappa}_2}{\bar{\kappa}_1}\right). \end{aligned} \tag{54}$$

All the restrictions in the above formulas are automatically fulfilled because of (46). The λ-integral in (53) transforms into

$$\kappa_1^{h-h_3-1}\kappa_2^{h-1}\int_0^1 dy\, y^{-h_3-1}(1-y)^{h-1}\left(1+\frac{\kappa_1}{\kappa_2}y\right)^{h-1}, \tag{55}$$

which is just an integral representation for the hypergeometric function (see, e.g., (15.3.1) in [14]) and yields

$$\frac{\Gamma(h)\Gamma(-h_3)}{\Gamma(h-h_3)}\kappa_1^{h-h_3-1}\kappa_2^{h-1} \times {}_2F_1\left(-h_3,1-h;h-h_3;-\frac{\kappa_1}{\kappa_2}\right), \tag{56}$$

provided (46) holds. The second contribution to (51), coming from the region $\bar\rho_2>\bar\rho_1$, is simply evaluated by the replacements $\bar\rho_1\leftrightarrow\bar\rho_2$, $\beta_1\leftrightarrow\beta_2$, which give

$$\int_{\bar\rho_2>\bar\rho_1} = (-1)^{n_3}\int_{\bar\rho_2<\bar\rho_1}\begin{pmatrix}\kappa_1\leftrightarrow\kappa_2\\ \bar\kappa_1\leftrightarrow\bar\kappa_2\end{pmatrix}, \tag{57}$$

where the parity factor stems from the change of sign of $\bar\rho_{12}$ in the power with exponent $-n_3$.

To derive the final expression for the Pomeron wave function in momentum space we have to analytically continue in the conformal weights to their physical values $h_3=h,\bar h_3=\bar h$ and $\kappa_y\in\mathcal{R}\iff\bar\kappa=\kappa^*$. To do this in (56) we use the relation (see (15.1.2) in [14])

$$\lim_{h_3\to h}\frac{1}{\Gamma(h-h_3)}\,{}_2F_1\left(-h_3,1-h;h-h_3;-\frac{\kappa_1}{\kappa_2}\right) = h(1-h)\frac{\kappa_1}{\kappa_2}\,{}_2F_1\left(1-h,2-h;2;-\frac{\kappa_1}{\kappa_2}\right). \tag{58}$$

Putting together (54,56,57,58), and rearranging some Γ-function factors according to the relation

$$\Gamma(\bar h)\Gamma(1-\bar h)=(-1)^n\Gamma(h)\Gamma(1-h) \qquad (h-\bar h=n\in\mathcal{N}), \tag{59}$$

we obtain the final expression

$$E^A_{h\bar h}(\kappa_1,\kappa_2)=\frac{h(1-h)\Gamma(1-h)\bar h(1-\bar h)\Gamma(1-\bar h)}{i^n\,(4\pi)^2} \times\left[\kappa_1^{*\,\bar h-2}\kappa_2^{h-2}\,{}_2F_1\left(1-h,2-h;2;-\frac{\kappa_1}{\kappa_2}\right)\times{}_2F_1\left(1-\bar h,2-\bar h;2;-\frac{\kappa_2^*}{\kappa_1^*}\right) + (-1)^n\{1\leftrightarrow 2\}\right]. \tag{60}$$

As a check, we show in the following that the above function is an eigenfunction of the Casimir operator of the Möbius group. In the coordinate representation, in complex notation, one has

$$\left((r_{12})^2\partial_1\partial_2+h(h-1)\right)\left(\frac{r_{12}}{r_1r_2}\right)^h=0, \tag{61}$$

together with a similar equation in the antiholomorphic variables. In the momentum representation, these equations read

$$\left((\partial_{\kappa_1}-\partial_{\kappa_2})^2\kappa_1\kappa_2+h(h-1)\right)E_{h\bar h}(\kappa_1,\kappa_2)=0, \tag{62}$$

and an analogous result holds for its antiholomorphic counterpart. From the general property of scaling invariance we know that

$$E_{h\bar h}(\kappa_1,\kappa_2)=\kappa_1^{h-2}\kappa_1^{*\,\bar h-2}E_{h\bar h}\left(1,\frac{\kappa_2}{\kappa_1}\right), \tag{63}$$

which is satisfied by (60). Changing the variables $\kappa_1\to p_1$, $\kappa_2/\kappa_1\to p_2$ transforms (62) into

$$\left(\left(\partial_{p_1}-\frac{1+p_2}{p_1}\partial_{p_2}\right)^2 p_1^2p_2+h(h-1)\right)\times p_1^{h-2}E_{h\bar h}(1,p_2)=0. \tag{64}$$

Taking the derivative with respect to p_1, we are left with a differential equation in the p_2 variable only. We represent it in terms of a new variable $y=-p_2$:

$$\left(y(1-y)\partial_y^2+(2-2(2-h)y)\partial_y-(h-1)(h-2)\right)\times E_{h\bar h}(1,-y)=0. \tag{65}$$

This is the well known hypergeometric equation. A linearly independent set of its solutions is given by the Kummer solutions u_1,u_4 (formulas (2.9.1) and (2.9.13) in [16]):

$$u_1(y)={}_2F_1(1-h,2-h;2;y)\quad, \qquad u_4(y)=(-y)^{h-2}\,{}_2F_1\left(1-h,2-h;2;\frac{1}{y}\right). \tag{66}$$

These solutions match exactly the structure in (60), which is therefore a solution of (62).

One can arrive directly at (60) by trying to construct a single-valued function from the two linearly independent solutions, and by further fixing the correct normalization. Note that δ-like terms do not appear in this approach, since we treat the holomorphic and antiholomorphic momenta as independent variables. The fact that, in principle, the two sectors do not simply commute (as one can see considering the two dimensional Poisson equation with a δ-like source) gives origin to the appearence of the δ-like terms.

Let us now consider the δ-like contributions. These are present when $\rho_1\to\infty$ or $\rho_2\to\infty$. Summing these two contributions one obtains

$$\tilde E^\delta_{h\bar h}(\boldsymbol{k}_1,\boldsymbol{k}_2)=\left[\delta^{(2)}(\boldsymbol{k}_1)+(-1)^n\delta^{(2)}(\boldsymbol{k}_2)\right]\times\frac{i^n}{2\pi}2^{1-h-\bar h}\frac{\Gamma(1-\bar h)}{\Gamma(h)}q^{\bar h-1}q^{*\,h-1}, \quad \boldsymbol{q}=\boldsymbol{k}_1+\boldsymbol{k}_2. \tag{67}$$

Appendix B: Integrals appearing in the coupling to the nucleon

In order to do the integral (37) we use the Feynman parametrization

$$\frac{1}{ABC} = 2\int_0^1 xdx \int_0^1 dy \frac{1}{D^3},$$

where

$$D = xyA + x(1-y)B + (1-x)C.$$

In our case

$$\begin{aligned} D &= xy(\boldsymbol{k}^2 + m^2) + x(1-y)((\boldsymbol{k}-\boldsymbol{l})^2 + m^2) \\ &\quad + (1-x)((\boldsymbol{k}-\boldsymbol{p})^2 + b^2) \\ &= (\boldsymbol{k} - x(1-y)\boldsymbol{l} - (1-x)\boldsymbol{p})^2 + R, \end{aligned}$$

where

$$\begin{aligned} R &= x(1-y)\boldsymbol{l}^2 + (1-x)(\boldsymbol{p}^2 + b^2) + xm^2 \\ &\quad - (x(1-y)\boldsymbol{l} + (1-x)\boldsymbol{p})^2. \end{aligned} \quad (68)$$

Shifting the integration momentum and performing the momentum integration we find the integral over x and y

$$I = \pi \int_0^1 xdx \int_0^1 dy \frac{1}{R^2}. \quad (69)$$

One of the integrations (say, of y) can be done analytically. We present

$$R(y) = \alpha + \beta y + \gamma y^2,$$

where

$$\begin{aligned} \alpha &= x(1-x)(\boldsymbol{l}-\boldsymbol{p})^2 + (1-x)b^2 + xm^2, \\ \beta &= x(2x-1)\boldsymbol{l}^2 + 2x(1-x)\boldsymbol{p}\cdot\boldsymbol{l}, \\ \gamma &= -x^2\boldsymbol{l}^2. \end{aligned}$$

The discriminant $\Delta = 4\alpha\gamma - \beta^2$ is negative, so that the integral over y gives

$$\begin{aligned} J &= \frac{\beta + 2\gamma}{\Delta R(1)} - \frac{\beta}{\Delta R(0)} + \frac{2\gamma}{\Delta\sqrt{-\Delta}} \\ &\quad \times \ln \frac{\beta + 2\gamma - \sqrt{-\Delta}}{\beta + 2\gamma + \sqrt{-\Delta}} \frac{\beta + \sqrt{-\Delta}}{\beta - \sqrt{-\Delta}}. \end{aligned} \quad (70)$$

One finds that as $x \to 0$ the integral J behaves like $1/x$, so that the integration over x is convergent around this point. As $x \to 1$, Δ is finite in the limit $m \to 0$. However, in the limit $x \to 1$ both $R(0)$ and $R(1)$ behave as $1/(1-x)$. So we have a logarithmic divergence at $x = 1$, regularized by finite m. Evidently only the first two terms in (70) lead to this divergence. Therefore we can safely put $m = 0$ in the third (logarithmic) term. In the vicinity of $x = 1$ we find

$$\begin{aligned} J \sim J_0 &= \frac{1}{\boldsymbol{l}^2}\left(\frac{1}{(1-x)(\boldsymbol{p}^2 + b^2) + m^2}\right. \\ &\quad \left. + \frac{1}{(1-x)[(\boldsymbol{l}-\boldsymbol{p})^2 + b^2] + m^2}\right). \end{aligned} \quad (71)$$

The final integration over x of this term will give

$$\begin{aligned} S &= \int_0^1 xdxJ_0 = \frac{1}{l^2}\left[\frac{1}{\boldsymbol{p}^2 + b^2}\left(\ln\frac{\boldsymbol{p}^2 + b^2}{m^2} - 1\right)\right. \\ &\quad \left. + \frac{1}{(\boldsymbol{l}-\boldsymbol{p})^2 + b^2}\left(\ln\frac{(\boldsymbol{l}-\boldsymbol{p})^2 + b^2}{m^2} - 1\right)\right]. \end{aligned} \quad (72)$$

One easily checks that the singular contributions coming from f_2 and f_3 are cancelled by the singular part of f_1 (34), so that the complete result is infrared finite.

To do the numerical calculation we present $J = J - J_0 + J_0 \equiv J_r + J_0$. The integral over x of the term $J_r = J - J_0$ converges at $x = 1$ and can be calculated numerically. The integral over x of J_0 is given by (72).

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